Assignment 11

Deadline: April 13, 2018.

Hand in: 9.1 no. 8, 13, Suppl. Ex. no 1a, 3ad.

Section 9.1: 8, 9, 11, 13

Supplementary Exercise

You should use the new definition of exponential, logarithmic, cosine and sine functions in the following problems.

1. Establish the following properties of the exponential and log functions: For every $\alpha > 0$,

   (a)
   $$\lim_{x\to\infty} \frac{x^\alpha}{e^x} = 0 \ ,$$

   (b)
   $$\lim_{x\to-\infty} |x|^\alpha e^x = 0 \ ,$$

   (c)
   $$\lim_{x\to\infty} \frac{\log x}{|x|^\alpha} = 0 \ ,$$

   (d)
   $$\lim_{x\to 0} |x|^\alpha \log |x| = 0 \ .$$

2. Establish the following properties of the cosine and sine functions:

   (a)
   $$\lim_{x\to 0} \frac{\sin x}{x} = 1 \ ,$$

   (b)
   $$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2} \ ,$$

3. Study the improper integrability of the following integrals:

   (a)
   $$\int_0^1 x^{-1/4} \log x \, dx,$$

   (b)
   $$\int_0^1 \frac{(1-\cos x)\log x}{x^2} \, dx \ ,$$

(c)

$$\int_0^\infty \frac{\sin x}{e^x - 1}\, dx \ ,$$

(d) Optional.

$$\int_0^\infty \frac{\sin x}{x}\, dx \ .$$

4. Optional. Consider $\sum_{n=1}^\infty a_n$ and let $\sum_{n=1}^\infty b_n$ and $\sum_{n=1}^\infty c_n$ where $b_n = a_n^+$ and $c_n = a_n^-$ (so $a_n = a_n^+ - a_n^-$). Show that $\sum_{n=1}^\infty b_n$ and $\sum_{n=1}^\infty c_n$ both are divergent to infinity when $\sum_{n=1}^\infty a_n$ is conditionally convergent.

5. Optional. Show that every conditionally convergent series admits a rearrangement which is divergent to infinity.